"ANALYSIS OF SOME DEVOLPED INVENTORY MODEL

WITH PRICE DEPENDENT DEMAND"

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Introduction-

Here discussing the limitations of traditional EOQ (Economic Order Quantity) models and the need for a more realistic approach that considers various factors such as demand rate variability, holding costs over time, and order size-dependent purchase costs. You also mention the objective of developing an inventory model that incorporates these factors and present a mathematical model for determining the optimal solution. The demand rate is not assumed to be constant but is instead influenced by variables like seasonality, selling price, and availability. The holding cost is also considered to be higher for longer storage periods, reflecting the reality that extended storage incurs additional costs. Furthermore, the unit purchase cost takes into account quantity discounts, acknowledging that larger order sizes often come with lower costs per unit.

To solve this model, a solution methodology is developed. It likely involves mathematical optimization techniques to determine the optimal order quantity and reorder points that minimize costs and optimize inventory levels. A numerical example is solved to illustrate the application of the model, and sensitivity analysis is conducted to examine the impact of different parameters on the optimal solution.This type of comprehensive inventory model aims to provide more accurate and practical insights into managing inventory levels, considering the dynamic nature of demand, holding costs, and purchase costs. By incorporating these factors, businesses can make better-informed decisions about ordering quantities and balancing inventory costs.

The growth of online transactions involving a hybrid cash-and-prepayment agreement during the current epidemic period. This agreement requires buyers to pay a portion of the purchase price in advance and the remaining amount upon delivery. Specifically, you are focusing on two sustainable inventory techniques for perishable goods: one with permitted shortages and fractional backlog and another without ending, taking into account a scenario where the buyer has a limited-capacity storehouse.Both inventory schemes demonstrate an increasing trend over time in terms of the demand

function and the decay rate of the perishable goods. However, the deterioration of the goods does not occur immediately but after a certain duration of storage. The decay rate also exhibits an upward trend with increasing storage time. To account for the cost of storing these goods, a linear function of storage time is adopted.

The objective is to determine when the inventory scheme without ending, considering salvage values, becomes more cost-effective for the industry manager compared to the inventory scheme with an ordinary selling price and an ending condition. A criterion is developed to make this determination.To validate the inventory methods, four numerical examples and a real-world case study are conducted. These analyses provide decision-making insights and conclusions for industry managers facing similar situations the investigating inventory management techniques for perishable goods under a hybrid cash-and-prepayment agreement, considering decay rates, storage costs, and decision-making criteria to optimize the industry manager's approach.

Basic Concept of Inventory-

A store's inventory is its whole stock of products and raw materials available at any one time for sale or manufacturing at the lowest possible cost. Stocked for the future are raw materials, commodities that are in the production process, finished goods, spare parts, etc. Materials inventory is a perfect resource; hence it is not intended for instant usage.

Inventory refers to the assortment of goods or materials that a business holds in preparation for future sales. It encompasses both the finished products ready for sale and the individual components or parts that are used in the production process.

Inventories play a vital role in the supply chain as they act as a bridge between the production of goods and their eventual sale to customers. They serve to fulfill customer demands by ensuring that products are available when needed. By maintaining appropriate levels of inventory, businesses can avoid stock outs and meet customer expectations in terms of product availability and delivery times.

Inventory management involves activities such as forecasting demand, determining optimal inventory levels, replenishing stock, and monitoring inventory turnover. Effective inventory management is crucial for balancing the costs associated with

carrying inventory, such as storage costs and the risk of obsolescence, with the benefits of having products readily available for sale.Inventories represent the stock of finished products and individual components that a company holds, serving as a connection between production and sales. Proper inventory management is essential for ensuring efficient operations and meeting customer demands.

Types of Inventory

Inventory is an idle resource with economic worth. It is a product that has been set aside or stored to meet potential demand. These things could be anything like resources, machinery, money, or even people. The following is a useful classification of the several types of inventory:

Raw materials

These are basic ingredients that have been held in stock and have not undergone any conversion in their receipt from the suppliers. Examples include lead, tin, cotton, rubber, lumber, and steel (angles, channels, flats, etc.), copper (sheet, tube, etc.), and lead.

Work-in-Progress inventories

These are the manufacturing materials that are only half finished (also known as semi-finished items or goods in process). Raw materials enter the work-in-process categorization at the conclusion of the first operation and stay there until they are transformed into finished items or piece parts. The materials in the trucks, conveyers, etc. are typically regarded as work in progress.

Spare parts inventories

The parts that are necessary for providing after-sales service to customers or for the full utilization of a machine by the buyer, then those parts would typically fall under the category of "spare parts" or "service parts" rather than inventory. Spare parts are components or items that are kept in stock by a company to support maintenance, repair, or replacement needs after a product has been sold. These parts are essential for addressing any potential issues or breakdowns that may occur during the product's lifecycle. By having spare parts available, companies can ensure prompt and efficient service to customers, minimizing downtime and maximizing customer satisfaction.

The parts needed for after-sales service or for the full utilization of a machine are typically categorized as spare parts or service parts, rather than being considered inventory in the traditional sense. These parts are crucial for providing customer support and ensuring the optimal performance of products or machines over their lifespan.

Inventory Terminology

1.Demand of a product is number of units required for fulfill the necessity in management of a business house. Amount of stock, which is to be maintained, depends upon the consumption or use of a commodity. It is of two types-

- **Deterministic demand pattern,**
- **Probabilistic or non-deterministic or stochastic demand pattern,**
- **Stock W.r.t price**
- **Time W.r.t price**
- **Price W.r.t price**
- **Ramp Type Demand**
- **Constant Demand**

2.Deterioration With the growth of time every natural thing changes its nature. The decrement in absolute of a product is deterioration.

The Certain goods, such as electronic devices, photographic film, grain, chemicals, pharmaceuticals, and other perishable items, can deteriorate over time, resulting in a gradual loss of their potential or utility. Deterioration can occur due to various factors such as exposure to environmental conditions, chemical reactions, natural decay, or obsolescence.

The deterioration of goods can be classified into two main types: constant and time-dependent.

1. Constant Deterioration: Constant deterioration implies that the rate of deterioration remains the same throughout the entire storage period. This means that the goods degrade at a constant rate, resulting in a linear reduction of their quality or utility over time.

2. Time-Dependent Deterioration: Time-dependent deterioration suggests that the rate of deterioration is not constant but varies with time. In this case, the goods may experience a slower rate of deterioration initially and then accelerate as more time passes. The deterioration process may follow a non-linear pattern, such as exponential decay, where the quality or utility declines more rapidly over time.

3.**Cost**

The capital which we are paying for the products is termed as cost. Cost is of several types depending upon different real-life mathematical scenarios.

4.**Effect of Learning**

There are several business houses in which same exercises repeated on fixed frequency. After every repeating time every associated person learn something to reduce the effective cost or to maximize the profit.

5.**Trade Credit**

Trade credit is an offer, offered by the seller to the buyer by which buyer takes benefit of delay in payments so that one can settle their accounts at a later stage to get the interest profit on capital for the delay period.

6.**Shortage/Backlogging**

If a product is not available on the demand of customer instantly this is termed as shortage of product.

7.**Inflation/Time Value of Money**

There is a natural phenomenon in real life to grow the things with time, this is applicable to inventory also, and from here the concept of inflation arises in inventory control system.

Model of Inventory-

Linear / Nonlinear Mathematical Models

Mathematical models are constructed using system parameters, which represent the relevant quantities of the system being analyzed. These parameters can be expressed as variables or constants and are manipulated using algebraic operators, functions, differential operators, or other mathematical operations.

In a mathematical model, the objective function and constraints define the relationships and limitations within the system. If both the objective function and constraints are represented entirely by linear equations, the model is termed as a linear model. Linear equations are mathematical expressions where the variables are raised to the power of 1 and are multiplied by coefficients, without any multiplication or division between variables.

Linear models have several distinguishing properties and advantages. The linearity of the equations allows for efficient mathematical analysis and optimization techniques, such as linear programming, to be applied. The solution space of linear models forms a convex set, making it easier to find optimal solutions and evaluate the impact of changes in parameters.

Deterministic / Probabilistic (Stochastic) Mathematical Model

Stochastic models differ from deterministic models in that they incorporate probabilistic elements. In stochastic models, variables are not described by single, deterministic values, but rather by probability distributions that represent the range of possible values and their associated probabilities.

In a stochastic model, uncertainty and randomness are explicitly considered, recognizing that the future outcomes or behavior of the system cannot be predicted with certainty. Instead, the model represents the likelihood or probability of different outcomes occurring. Stochastic models are particularly useful when dealing with systems that exhibit inherent randomness, variability, or uncertainty. They are commonly used in various fields, such as finance, operations research, queuing theory, and simulation.

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Operations Research Foundations and Background

The methods of OR can be used in inventory control, waiting line problems, machinery failure prediction, human resource management, population science, queening theory, management science, production management, industrial engineering, etc.

Hypothesis-

1. The efficient market hypothesis states that share prices in the stock market reflect all available information.

2. Stock prices on exchanges are generally considered to trade at their fair market value, determined by the interactions of buyers and sellers in the marketplace.

3. Proponents of passive portfolio management argue that investors can benefit from investing in a low-cost, passive portfolio that aims to replicate the performance of a specific market index.

4. Opponents of passive portfolio management believe that it is possible to outperform the overall market by employing active management strategies.

Review of Literature

A researcher can always make his research that addresses problems relevant to society is crucial for generating fruitful outcomes. When researchers focus on real-world problems and draw insights from existing literature, they can contribute to the advancement of knowledge and provide practical solutions.
To achieve this, it is essential for researchers to review past problems and solutions in the inventory theory domain. By examining the existing body of literature, researchers can gain a comprehensive understanding of the topic, identify gaps or unresolved issues, and build upon previous findings.

Ghare and Schrader [1963] introduced the concept of deterioration in inventory models with, “A model for exponentially decaying inventory”, in this model they assumed constant price as well as constant rate of deterioration. Gupta and Vrat [1986] initiated the stock w.r.t price in inventory models with inventory model for stock dependent consumption rate; in this model they took the price dependent on stock levels. Datta and Pal [1993] added shortage in their deterministic inventory

model for deteriorating items with inventory-level-dependent price rate. Sarma and Naidu [1995] gave an EOQ model with two levels of storage when the quantity received is uncertain. Balkhi and Benkherouf researched on an inventory model for deteriorating items with stock dependent and time varying price rates. Sana and Chaudhary [2004], wrote on a volume flexible production policy for a deteriorating item with time w.r.t price and shortages. Teng et al. [2007], developed retailer's optimal ordering policies with trade credit financing. Chern et al. [2008], developed partial backlogging inventory lot-size models for deteriorating items with fluctuating demand under inflation. Fei and Dongyi [2010], developed optimal replenishment policy for the EPQ model with permissible delay in payments and allowable shortages. Again Singh et al. developed an inventory model with time dependent price rates under the effect of inflation. Mishra and Singh [2011], gave deteriorating inventory model for time w.r.t price and holding cost with partial backlogging. Tripathi and Padhanb [2012], published an EOQ model for three parameter Weibull deterioration with permissible delay in payments and associated salvage value. Guria et al. [2013], developed inventory policy for an item with inflation induced purchasing price, selling price and demand with immediate part payment. Yadav et al. [2013], studied retailer's optimal policy under inflation in fuzzy environment with trade credit. Amutha and Chandrasekaran [2013], developed an inventory model for deteriorating items with three parameter weibull deterioration and price w.r.t price. Agrawal [2013] et al. developed inventory model with deteriorating items, ramp-type demand and partially backlogged shortages for a two-warehouse system. Tayal et al. [2014], developed an inventory model for deteriorating items with seasonal products and an option of an alternative market. Again, Tayal et al. [2014], created two echelon supply chain model for deteriorating items with effective investment in preservation technology. Patel and Parekh [2014] gave inventory model for variable deteriorating items with two warehouses under shortages, time varying holding cost, inflation and permissible delay in payments. Dutta and Chauhdhary [2015] provided a Fuzzy based two warehouse inventory models for deteriorating items with cubic demand and different fuzzy cost parameters. Khurana [2015] gave two-warehouse inventory model for deteriorating items with time w.r.t price under inflation. Yadav [2016] et al. established two-warehouse inventory model for deteriorating items with variable holding cost, time-w.r.t price and shortages. Dave and Patel, Elsayed and Terasi and Sachan worked on (T, Si) policies in inventory model for deteriorating items with

time proportional price. Sarkar and Saran in [2001], gave an inventory model with trade credit policy and variable deterioration for fixed lifetime products.

Research Gaps-

Research Gaps would Identified after doing this critical survey of present state of knowledge, to know that some topics still require more improvement to get better and efficient results by inventory modeling. Progress is continued day by day, by this critical review, concluded that there is always a scope of new research in inventory models with price market scenario.

1. By our investigations for repeating same type of works again we found the need of development of a model for deteriorated items with stock w.r.t price and learning effect, here learning effect provides a new part of knowledge to repeat the same process.

2. To develop an inventory model for non-instantaneous deteriorating products with a selling price-dependent price rate under the learning effect. Products that have a fixed period of deterioration-free shelf life. Considering the non-instantaneous deterioration of products implies that their quality or usefulness gradually declines over time, rather than deteriorating instantly. This is a relevant factor to consider in inventory management, as it affects the timing of orders and the determination of optimal inventory levels.

From present state of knowledge and market survey researcher will collected different problems to be solve, and would concluded to develop the above mention models identified as research gaps.

Proposed study-

The research objective of developing mathematical models to minimize the annual total cost of inventory is a valuable endeavor for inventory management, particularly in the context of a medicine stockist facing specific constraints. By incorporating various factors such as deterioration, stock dependence on price, the learning effect, and inflation, the study aims to create models that can optimize inventory decisions and reduce overall costs.

Objectives-

1. To be analysis of inventory model for deteriorated items with stock w.r.t price and demand.

2. To be analysis of inventory model with stock and selling price dependent price rate, partial backlogging and variable ordering cost.

3. To be analysis of inventory model for non-instantaneous deteriorating products with selling price dependent price rate under learning effect.

4. To be analysis of production inventory model with stock w.r.t price and price shortages under inflationary environment.

5. To be analysis of inventory model for deteriorated items with exponential demand under trade credit.

Methodology:

1. Literature survey.
2. Model development.
3. Development of computer programs.
4. Real time simulation of models developed.
5. Evaluation of the results.
6. Compilation of the final results, theories developed/evolved and preparation of the thesis.

BIBLOGRAPHY-

[1] P. M. Ghare and S. F. Schrader, “A model for exponentially decaying inventory”, *Journal of Industrial Engineering*, 14, pp. 238–243, 1963.

[2] W. A. Donaldson, “Inventory replenishment policy for a linear trend in demand-an analytical solution”, *Operational Research quarterly*, 28(3), pp. 663-670, 1977.

[3] R. Gupta, and P. Vrat. "Inventory model for stock-dependent consumption rate." *Opsearch* 23.1 (1986): 19-24.

[4] S. Pal, A. Goswami and K. S. Chaudhuri, “A deterministic inventory model for deteriorating items with stock-dependent demand rate*”, International Journal of Production Economics*”, 32, pp. 291–299, 1993.

[5] S. Sana and K. S. Chaudhary, “On a volume flexible production policy for a deteriorating item with time dependent demand and shortages”, *Advanced Modeling and Optimization*”, 6(1), pp. 57-74, 2004.

[6] J. T. Teng, C.T. Chang, M.S. Chern and Y.L. Chan, “Retailer’s optimal ordering policies with trade credit financing”, *International Journal of Systems Science*”, 38, pp. 269– 278, 2007.

[8] M. S. Chern, H. L. Yang, J. T. Teng and S. Papa Christos, “Partial backlogging inventory lot-size models for deteriorating items with fluctuating demand under inflation”, *European Journal of Operational Research*”, 191, pp. 127–141, 2008

[9] Hu, Fei, and Dongyi Liu. "Optimal replenishment policy for the EPQ model with permissible delay in payments and allowable shortages." *Applied Mathematical Modelling* 34.10 (2010): 3108-3117.

[10] V. K. Mishra and L. S. Singh, “Deteriorating inventory model for time dependent demand and holding cost with partial backlogging”, *International Journal of Management Science and Engineering Management*, 6(4), pp. 267-271, 2011.

[11] C. K. Tripathi and L.M. Padhanb, “An EOQ model for three parameter Weibull deterioration with permissible delay in payments and associated salvage value”, *International Journal of Industrial Engineering Computations*, 3, pp. 115–122, 2012.

[12] A. Guria, et al. "Inventory policy for an item with inflation induced purchasing price, selling price and demand with immediate part payment." *Applied Mathematical Modelling* 37.1-2 (2013): 240-257.

[13] D. Yadav, S. R. Singh and R. Kumari, “Retailer’s optimal policy under inflation in fuzzy environment with trade credit”, *International Journal of System Science*, 44, pp. 1-9, 2013.

[14] S. Agrawal, Snigdha Banerjee, and Sotirios Papachristos. "Inventory model with deteriorating items, ramp-type demand and partially backlogged shortages for a two-warehouse system." *Applied Mathematical Modelling* 37.20-21 (2013): 8912-8929.

[15] R. Amutha and E. Chandrasekaran, "An inventory model for deteriorating items with three parameter Weibull deterioration and price dependent demand", *International Journal of Engineering Research & Technology*, 2(5), pp. 931-935, 2013

[16] R. Akter and Y. Chung, "An Evolutionary Approach for Document Clustering", *International Conference on Electronic Engineering and Computer Science Procedia*, 4, 370-375, 2013.

[17] S. Tayal, S. Singh, and R. Sharma. "An inventory model for deteriorating items with seasonal products and an option of an alternative market." *Uncertain Supply Chain Management* 3.1 (2015): 69-86.

[18] S. Tayal, S. R. Singh, R. Sharma and A. Chauhan, "Two echelon supply chain model for deteriorating items with effective investment in preservation technology", *International Journal Mathematics in Operational Research*, 6(1), pp. 78-99, 2014.

[19] R. Patel and R. U. Parekh, "Inventory model for variable deteriorating items with two warehouses under shortages, time varying holding cost, inflation and permissible delay in payments", *International Refereed Journal of Engineering and Science*, 3(8), pp. 06-17, 2014.

[20] D. Khurana, "Two warehouse inventory model for deteriorating items with time dependent demand under inflation", *International Journal of Computer Applications*, 114(7), pp. 34-38, 2015

[21] P. Dutta and P. D. Chauhdhary, "A fuzzy based two warehouse inventory model for deteriorating items with Cubic Demand and different fuzzy cost parameters", *International Journal of Engineering Research and General Science*, 3(5), pp. 209-218, 2015.

[22] A. S. Yadav, A. Swami, S. Kumar and R. K. Singh, "Two-warehouse inventory model for deteriorating items with variable holding cost, time-dependent demand and shortages", *IOSR Journal of Mathematics*, 12(2), pp. 47-53, 2016.

[23] Bones, Christian C., Luciana AS Romani, and Elaine PM de Sousa. "Improving multivariate data streams clustering." *Procedia Computer Science* 80 (2016): 461-471.

[24]. R. P.Tripathi, and S. Aneja. "Inventory models for stock-dependent demand and time varying holding cost under different trade credits." *Yugoslav Journal of Operations Research* 28.1 (2017).

[25]. Vandana. "Analysis of an Inventory Model with Time-dependent Deterioration and Ramp-type Demand Rate: Complete and Partial Backlogging." *APPLICATIONS AND APPLIED MATHEMATICS-AN INTERNATIONAL JOURNAL* 13.2 (2018): 1076-1092.

[26]. P. Sharma, et al. "Effect of inflation on two storage inventory model with time dependent deteriorating items and stock dependent demand." *International Journal of Mathematical, Engineering and Management Sciences* 5.3 (2020): 544.

[27]. BA. Kumar, Susanta Kumar Paikray, and Umakanta Mishra. "Two-storage fuzzy inventory model with time dependent demand and holding cost under acceptable delay in payment." *Mathematical Modelling and Analysis* 25.3 (2020): 441-460.

[28]. Kusum and V. Kumar, "A model for deterministic inventory with deteriorating items with demand dependent on time fractionally and constant holding and cost deterioration rate," *International Journal of Mathematics Trends and Technology*, (2021).

[29]. C. K. Sivashankari, and Lalitha Ramachandran. "Inventory models with integrated time dependent demands for deteriorating items–in third and fourth order equations." *RAIRO--Operations Research* 55.5 (2021).

[30]. MAA. Khan, et al. "Inventory management with hybrid cash-advance payment for time-dependent demand, time-varying holding cost and non-instantaneous deterioration under backordering and non-terminating situations." *Alexandria Engineering Journal* 61.11 (2022): 8469-8486.